Informatics in Pāṇini's Grammar

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Outline

Structure of Aṣṭādhyāyī

Analysis of Sanskrit Language

> *The mathematical method is characteristic of much of Western philosophy whereas the grammatical method is characteristic of much of Indian philosophy.*

Daniel H H Ingalls, 'Comparison of Indian and Western Philosophy', Journal of Oriental Research, (2),1954.

Informatics

Etymology: Informatics

German:Informatik

French:Informatique

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Information + auto**matique** ⇒ informatique
Informatics: Science of Automatic processing of Information
Information coding device: Language

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Computational Linguistics ⊂ Informatics

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The First Formal system of Grammar: Pāṇini's Aṣṭādhyāyī

Pāṇini's Aṣṭādhyāyī

Circa 500 B.C.E.

Extant Grammar of the then prevalent Sanskrit Language

Around 4000 sutras[1](aphorism)

8 chapters 4 sections each

[1] alpākṣaram(concise) asandigdham(unambiguous) sāravat(contains essence) viśvatomukham(general) | astobham(continuous) anavadyam(without flaw) ca sūtraḥ sūtravido viduḥ||(vāyu purāṇa)

Aṣṭādhyāyī .. contd

It is admired for its

- simplicity
- completeness of the descriptive coverage
- rigorous consistency in the use of meta language
- intricate system of conventions governing rule application and rule interaction
- richness in various aspects of informatics

One of the greatest monuments of human intelligence (Bloomfield) is only beginning to claim its rightful position in linguistics. Many of the insights of Panini's grammar still remain to be recaptured, but those that are already understood constitute a major theoretical contribution.

Paul Kiparsky, Emeritus Prof. Stanford University
'The encyclopedia of Language and Linguistics', Asher, pp 2923.

Panini, then, was not an ancient and nebulous precursor of a science in which everything has since been done better, but a distant colleague of genius from whom linguists are still able to learn.

Prof. Fritz Staal
Pāṇini , 'Encyclopedia of Languge and Linguistics', Vol-6, Page no: 2917; Ed: Asher

Not only Panini was by far the first linguist in recorded history, but I claim he was the first informaticien, 24 centuries before computers came into existence.

– Prof. Gérard Huet, Computer Scientist, Inria, Paris, in the Inaugural Speech at the First International Sanskrit Computational Linguistics Symposium, Paris, 29th october, 2007

Informatics in Aṣṭādhyāyī

From informatics point of view,
the importance of Aṣṭādhyāyī is two fold,

- The Structure of Aṣṭādhyāyī
- Method of Analysis of Sanskrit Language

Aṣṭādhyāyī

Aṣṭādhyāyī consists of around 4000 aphorisms with some ancillary texts.

- śivasūtras (special order of the phonemes)
- dhātupāṭha (list of verbal roots)
- gaṇapāṭha (various sets of nouns)
- liṅgānuṣāsanam (system for deciding the gender)
- uṇādi sūtras (?) (special rules)

śivasūtrāṇi

Normal Arrangement of Alphabet

Vowels	a	ā	i	ī	u	ū	ṛ	ṝ	ḷ	e	ai	o	au	aṃ	aḥ
Velar	ka	kha	ga	gha	ṅa										
Palatal	ca	cha	ja	jha	ña										
Retroflex	ṭa	ṭha	ḍa	ḍha	ṇa										
Dental	ta	tha	da	dha	na										
Labiel	pa	pha	ba	bha	ma										
semi-vowel	ya	ra	la	va											
Fricative	śa	ṣa	sa	ha											

śivasūtrāṇi

Pāṇini required several(42) subsets of this alphabet to describe various operations.

śivasūtrāṇi

Some of these subsets:

All vowels
All consonants
All vowels + semivowels + nasals
The voiceless stops
stops and sibilants
Any of *ba ga ḍa da*
Any of *bha gha ḍha dha*
Any of *ya va ra la ña ma ṅa ṇa na ma jha bha*
....

śivasūtrāṇi

It is not advisable to give 42 names to these sets.
It will be difficult to memorize the association.

These are Partially ordered sets.

Pāṇini arranged them linearly in the form of 14 ShivasUtras.

śivasūtrāṇi

1. *a i u (Ṇ)*
2. *ṛ ḷ(K)*
3. *e o (Ṅ)*
4. *ai au (C)*
5. *ha ya va ra(Ṭ)*
6. *la (Ṇ)*
7. *ña ma ṅa ṇa na (M)*
8. *jha bha(Ñ)*
9. *gha ḍha dha (Ṣ)*
10. *ja ba ga ḍa da (Ś)*
11. *kha pha cha ṭha tha ca ṭa ta (V)*
12. *ka pa (Y)*
13. *śa ṣa s (R)*
14. *ha (L)*

śivasūtrāṇi

Justification of this arrangement is
attempted independently by

Cardona (on historical grounds)
Stall (linguistically)
Kiparsky (logically)
and Petersen (mathematically)

Petersen (2008) proved that the arrangement is optimal.
And is one among the 12 000 000 possibilities.

śivasūtrāṇi

The set *khar*

1 *a i u (Ṇ)*
2 *ṛ ḷ(K)*
3 *e o (Ṅ)*
4 *ai au (C)*
5 *ha ya va ra(Ṭ)*
6 *la (Ṇ)*
7 *ña ma ṅa ṇa na (M)*
8 *jha bha(Ñ)*
9 *gha ḍha dha (Ṣ)*
10 *ja ba ga ḍa da (Ś)*
11 *kha pha cha ṭha tha ca ṭa ta (V)*
12 *ka pa (Y)*
13 *śa ṣa s (R)*
14 *ha (L)*

śivasūtrāṇi

1 *a i u (Ṇ)*
2 *{r ḷ}(K)*
3 *{{e o} (Ṅ)*
4 *{ai au}} (C)*
5 *ha ya va ra(Ṭ)*
6 *la (Ṇ)*
7 *ña ma {ṅa ṇa na} (M)*
8 *jha bha(Ñ)*
9 *{gha ḍha dha} (Ṣ)*
10 *ja {ba ga ḍa da} (Ś)*
11 *{kha pha} {cha ṭha tha} {ca ṭa ta} (V)*
12 *{ka pa} (Y)*
13 *{śa ṣa s} (R)*
14 *ha (L)*

$2! * 2! * 2! * 2! * 3! * 3! * 4! * 2! * 3! * 3! * 2! * 3! \approx 12\,000\,000$

śivasūtrāṇi

Given a set of Partially Ordered Sets,

Now it is possible to tell

Whether the elements are

Śivasūtra encodable or not.

Weibke Petersen(2008)

Structure of Aṣṭādhyāyī

A) The First Formal Grammar

a) A Formal Grammar is written in a Formal Language.
b) The Formal language has well-defined Syntax.

Aṣṭādhyāyī is written in Sanskrit.
The syntax as well as the programme are intermixed in the same piece of work.

Formal Grammar

$G = (N, \Sigma, P, S)$

N : A finite set of Non-terminals

S: The Start Symbol $\in N$

Σ: A finite set of Terminals

P: Production rules, of the type
$(\Sigma \cup N) * N(\Sigma \cup N)* \rightarrow (\Sigma \cup N)*$

Syntax of Aṣṭādhyāyī

Example 1

padam(word)::=	*subantam*(nominal form)
|	*tiṅantam*(verbal form)
(suptiṅantam padam 1.4.14)	
;	
subantam (nominal form)::=	*prātipadikam* (nom stem)*sup*(nom suff)
prātipadikam(nominal stem)::=	*kṛt* (noun derived from a verb)
(*kṛttaddhitasamāsāśca* 1.2.46)	
|	*taddhita*(noun derived from a noun)
|	*samāsa*(compound)
|	underived_prātipadikam(nominal stem)
;	
samāsa(compound)::=	*alaukika vigrahaḥ*(Intermediate ling exp)
;	
alaukika vigrahaḥ::=	*prātipadikam sup prātipadikam sup*
(*saha supā* 2.1.4)	
;	

Syntax of Aṣṭādhyāyī

```
A ::=   B
      | C
      ;
B ::=   D E
D ::=   F
      | G
      | H
      | I
      ;
H ::=   J
      ;
J ::=   K '+' L M '+' N
      ;
```

Syntax of Aṣṭādhyāyī

Example 2:
iko yaṇaci (6.1.77)

ik{6} yaṇ{1} ac{7}

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tasmin iti nirdiṣṭe pūrvasya (1.1.65)
A word ending in locative case indicates 'of the preeceding'.

ṣaṣṭhī sthāneyogā (1.1.48)
A word in genitive case undergoes change.

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ik{6} *ac*{7} ::= *yaṇ*{1} *ac*{7}
{i,u,ṛ,ḷ} {a,i,u,ṛ,ḷ,e,o,ai,au} → {y,v,r,l} {a,i,u,ṛ,ḷ,e,o,ai,au}

Syntax of Aṣṭādhyāyī

tasmin iti nirdiṣṭe pūrvasya (1.1.65)
A word ending in locative case indicates 'of the preceeding'.

tasmāt iti uttarasya (1.1.66)
A word ending in ablative case indicates 'of the following'.

ṣaṣṭhī sthāneyogā (1.1.48)
A word in genitive case undergoes change.

W{5} W{6}W{7} ::= W{5} W{1}W{7}

Ingermann observed that the sūtras of Aṣṭādhyāyī have the same structure as that of BNF and suggested to rename 'Backus Naur Form' as 'Pāṇini Backus Naur Form' (1967 ACM Communications).

Anuvṛtti

Distributivity in Mathematics:
a * (b + c) = a * b + a * c

Distributivity in Languages:

John goes home.
John eats fruits.

John goes home and eats fruits

Or simply,
John goes home,
eats fruits.

Anuvṛtti

Consider the following sūtras:

upadeśe ac anunāsik it 1.3.2

hal antyam 1.3.3

na vibhaktau tusmāḥ 1.3.4

ādiḥ ñiṭuḍavāḥ 1.3.5

ṣaḥ pratyayasya 1.3.6

cuṭū 1.3.7

laṣaku ataddhite 1.3.8

Anuvṛtti

upadeśe it

ac anunāsik (=it)

hal antyam

na vibhaktau tusmāḥ (=it)

ādiḥ

ñiṭuḍavāḥ (=it)

pratyayasya

ṣaḥ (=it)

cuṭū (=it)

laśaku (=it) ataddhite

Anuvṛtti

upadeśe (a) *it* (c)

ac anunāsik (= it)(b)

hal antyam (d)

na vibhaktau tusmāḥ(=it) (e)

ādiḥ (f)

ñiṭuḍavāḥ (=it) (g)

pratyayasya (h)

ṣaḥ(=it) (i)

cuṭū (=it) (j)

laśaku (=it) ataddhite (k)

Anuvṛtti

a c

b

d

e

f

g

h

i

j

k

a (b + de + f [g + h { i + j + k }]) c

Anuvṛttiḥ

No Proper Nesting; *maṇḍūka plutiḥ*

1.1.1 *vṛddhiḥ ādaic*

1.1.2 *adeṅ guṇaḥ*

1.1.3 *ikaḥ guṇavṛddhī* (*vṛddhiḥ guṇaḥ*)

Anuvṛttiḥ

Maximum advantage of features of Natural Language:

How are the complete phrases reconstructed?

ākāṅkṣāḥ (Expectancy): Major role in deciding the *anuvṛtti* (Bhate)

Anuvṛttiḥ

Example of borrowing from as many as 12 sūtras

Original sūtra: 3-3-65 *kvaṇaḥ vīṇāyām ca*
After *Anuvṛtti*: pratyayaḥ paraḥ ca (ādyudāttaḥ ca dhātoḥ kṛt kriyārthāyām bhāve akartari ca kārake sajñāyām) ap upasarge vā nau

Anuvṛttiḥ

Some Statistics:

Total Sūtras	(3,984) 4,000
Total Words	(7,007) 7,000
Total Words after repeating the words with anuvṛtti	40,000
Compression due to anuvṛtti	(40,000/7,000 $\approx$) 1/6
In terms of byte size, compression	1/3

Significant from Oral Tradition.
The time to memorise the sūtras grows exponentially.
With anuvṛtti, a student can memorise the complete Aṣṭādhyāyī in about only 6 months!

Ordering of the rules

pūrvatra asiddham (8.2.1)

asiddhavat atra ābhāt (6.4.22)

ṣatvatukorasiddhaḥ (6.1.83)

Ordering of the rules

asiddhavat atra ābhāt (6.4.22)

hujhulbhyoḥ herdhiḥ (6.4.101)

śāhau (6.435)

6.4.101: *śās* + *hi* → *śās dhi*

6.4.35: *śās* + *hi*→ *śā* + *hi*

Application of one rule blocks the application of the other.
Both the rules need to be applied.

asiddhavat atra ābhāt (6.4.22) → Parallel Computing

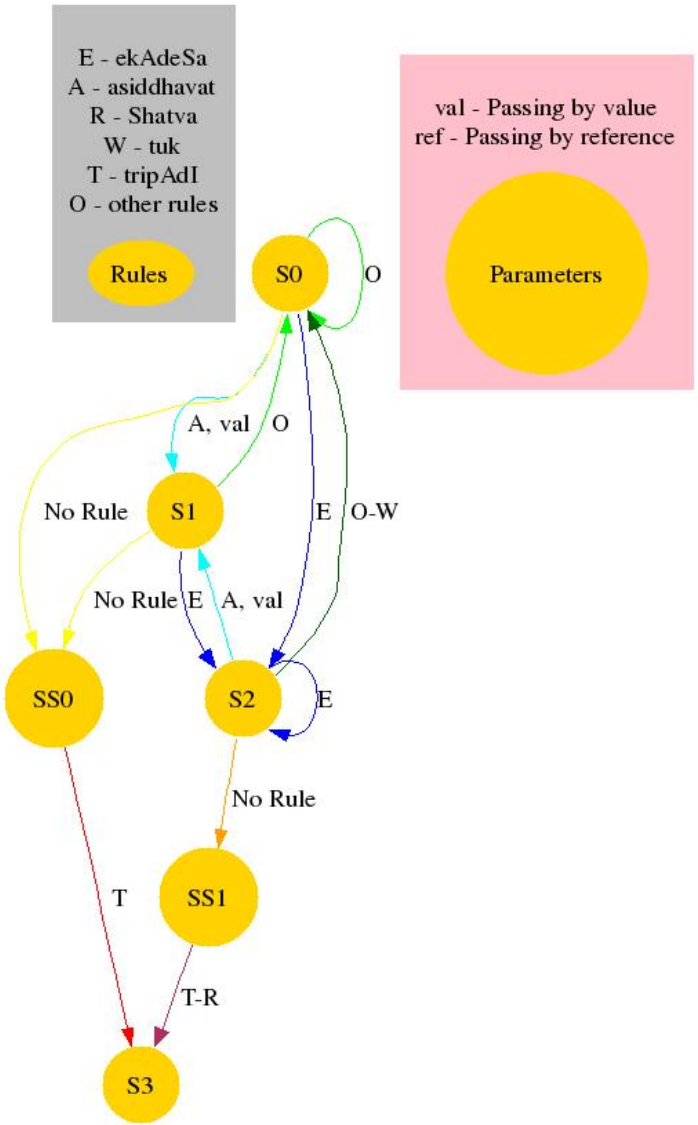
Conflict Resolution

Programming Languages

- Longest match
- Precedence Rules

Aṣṭādhyāyī

- *vipratiṣedhe paraṃ kāryam* (1.4.2)


E - ekAdeSa
A - asiddhavat
R - Shatva
W - tuk
T - tripAdI
O - other rules
Rules
val - Passing by value
ref - Passing by reference
Parameters
S0
O
A, val
O
No Rule
S1
E
O-W
No Rule
E
A, val
SS0
S2
E
No Rule
T
SS1
T-R
S3

Modularity

The rules related to a particular task are grouped together.
For example,
consider the following sūtras which identify sounds used as markers (*anubandha*).

upadeśe ac anunāsik it 1.3.2

hal antyam 1.3.3

na vibhaktau tusmāḥ 1.3.4

ādiḥ ñiṭḍavāḥ 1.3.5

ṣaḥ pratyayasya 1.3.6

cuṭū 1.3.7

laṣaku ataddhite 1.3.8

Features of Aṣṭādhyāyī .. contd

Object Oriented Programming:

Encapsulation of data with the (markers to the) functions
Bhaj + (gh)a(ñ): In the presence of gh, j $\rightarrow$ g

Inheritance:

Multiple inheritance $\rightarrow$ arranged as a linear inheritance
derivational suffixes, deriving a noun from a noun (Taddhita pratyaya)
(Ashwini Deo 2007)

Analysis of Sanskrit Language

Pāṇini paid utmost attention to the dynamics of Information flow while analysing Sanskrit.

We cite 3 sūtras to highlight this point.

anabhihite(1.3.1) (*Where* is the information Coded)

svatantraḥ kartā (1.4.54) (*How much* information is coded)

samānakartṛkayoḥ pūrvakāle (3.4.21) (*How* is the information coded)

anabhihite (Where is the information coded?)

Skt: rāmaḥ rāvaṇam hanti.
Gloss: Rama{nom} Ravana{acc} kill{3p sg pres}
Eng: Rama killed Ravana.

Skt: rāmeṇa rāvaṇaḥ hataḥ.
Gloss: Rama{inst} Ravana{nom} kill{3p sg past pl}
Eng: Ravana was killed by Rama.

svatantraḥ kartā (1.4.54) (*How much* information is coded)

John opened the door with a key.

This key opened the door.

The wind opened the door.

The door opened.

svatantraḥ kartā (1.4.54) (*How much* information is coded)

John opened the door with a key.

This key opened the door.

The wind opened the door.

The door opened.

John opened the door with a key. Agent

This key opened the door. Instrument

The wind opened the door. Cause

The door opened. goal

In Pāṇinian Framework, all of them are **kartā**!

Greatness of *Pāṇini* lies in
identifying EXACTLY HOW MUCH
information is coded in a language string.
$\Rightarrow$ Upper Bound for the possible Analysis
using only a language string and grammar.

We can extract precisely only that information which can be decoded from the language string 'without any requirement of additional knowledge'.

Analogy:
We can not do high quality work with low quality energy.

Dhanyavādaḥ

Thank you for your attention